Spring 2025

The University of Texas at Austin

Machine Learning WB CS391L

Lecture 9 - Introduction to Deep Learning

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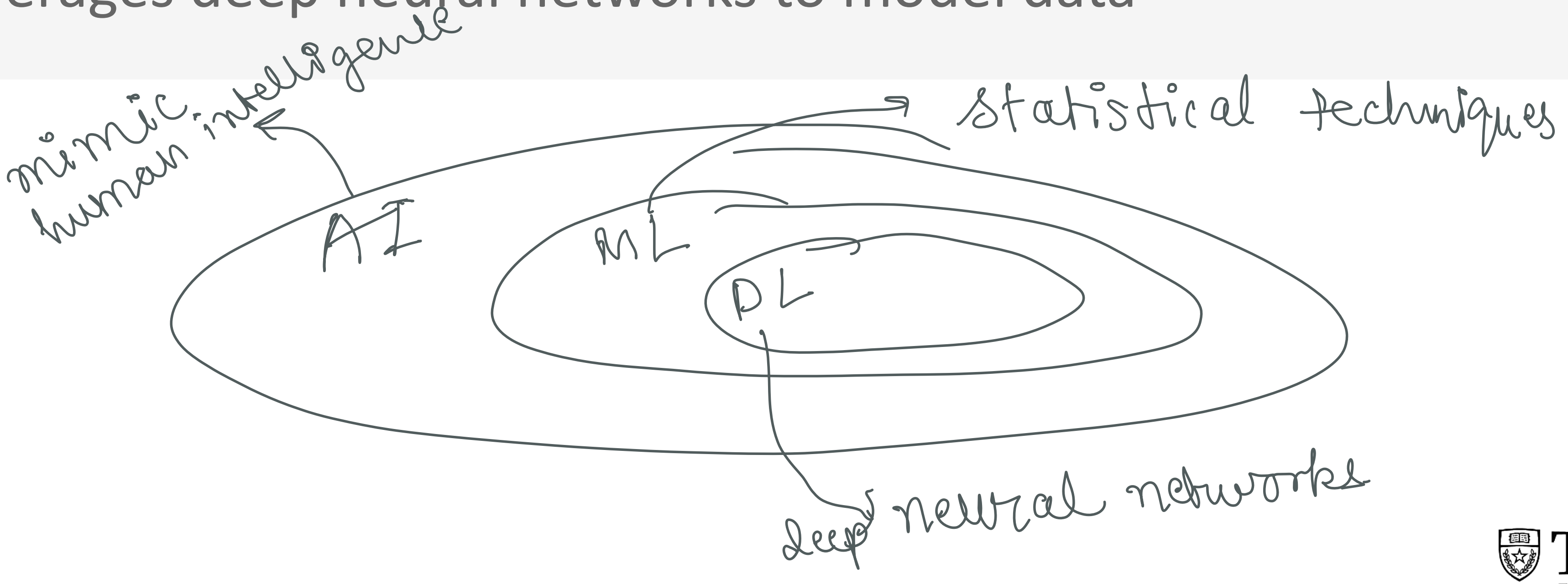
What is Deep Learning?

Deep Learning (DL) is a subfield of Machine Learning (ML) that leverages deep neural networks to model data



Why Deep Learning?

Automatically learn representations from data, eliminating the need for manual feature engineering

Deep Learning - Enablers

Hardware - GPUs happened!

Data - Internet happened!

Frameworks - Pytorch, Tensorflow, JAX happened!

Deep Learning - Applications

Any intelligent task!

- Object recognition
- Self driving
- Playing games
- Conversation agent
- Media (image/video) generation
- …
- Artificial general intelligence?

Deep Learning - Key Components

Model - a "neural network" to map input data to output prediction

Loss - a scalar that quantifies how well a model fits our data

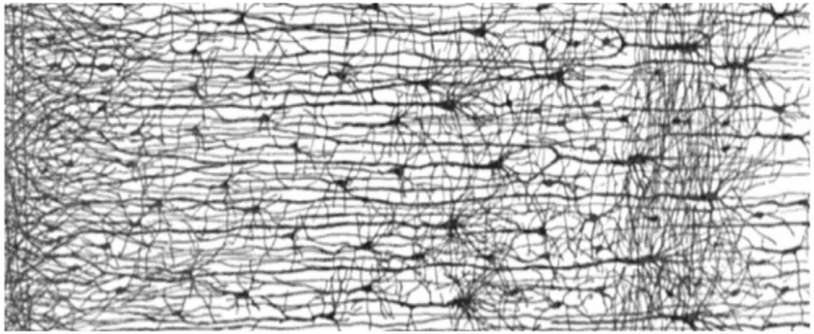
Optimization - a process (typically "gradient descent") to adjust the model parameters to minimize the loss

Model

Neural Networks

Neural Networks - Brief History

- Early work on perceptrons in the 1950s laid the foundation
- Modeled after the human brain's neurons



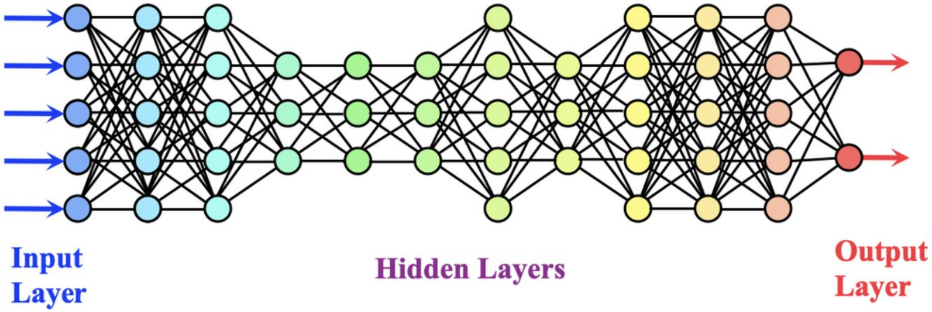


Image source - https://mriquestions.com/what-is-a-neural-network.html

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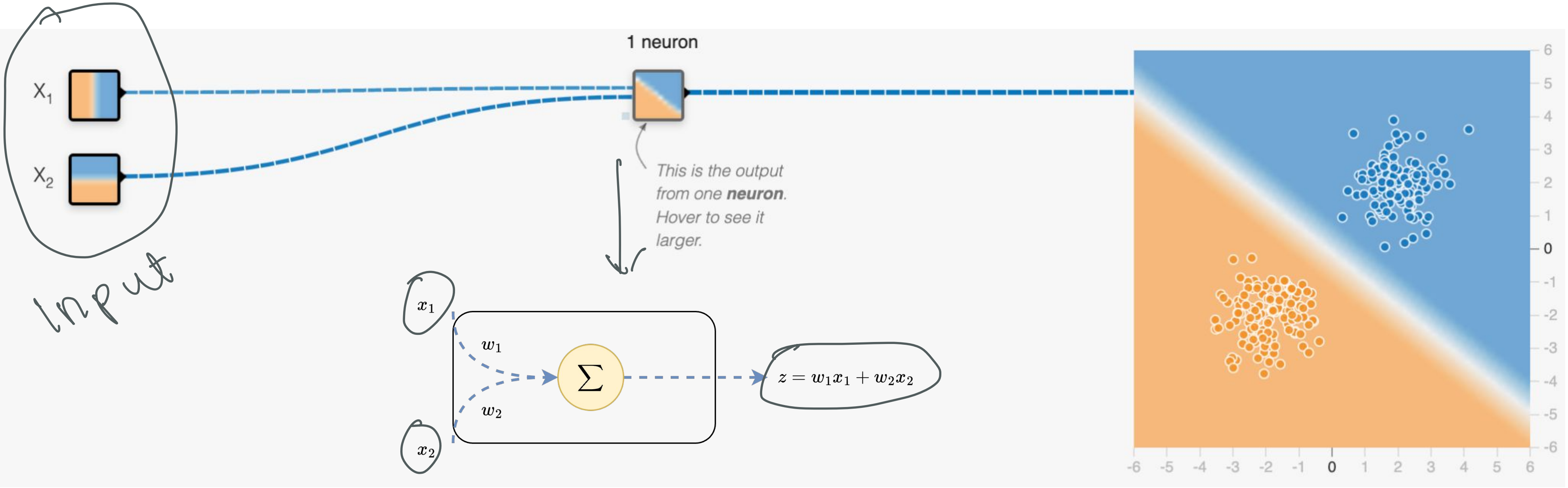
Neural Networks - Building Block

Neuron/Perceptron a basic computational unit

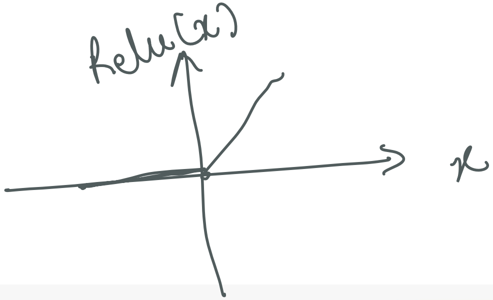
- **Inputs** - Receives multiple scalar inputs
- **Weights** - Each input has a weight, importance of that input
- **Summation** - The neuron adds up all the weighted inputs
- **Non-linearity** - A filter/activation function

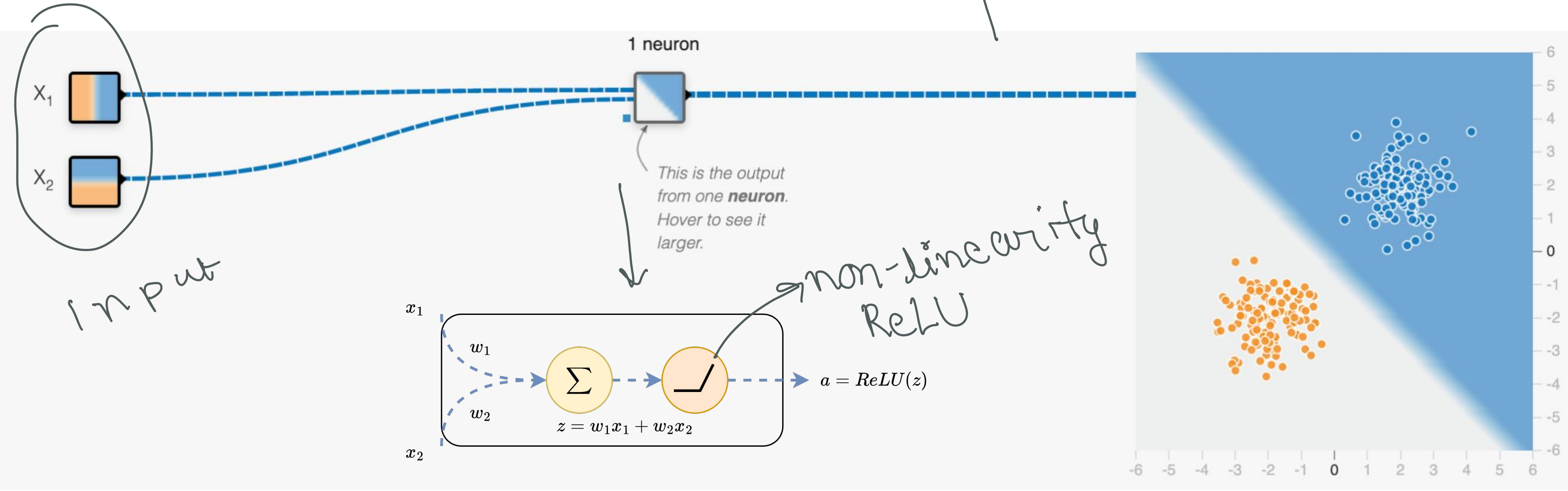
Linear Neuron



https://playground.tensorflow.org/

Non-linear Neuron





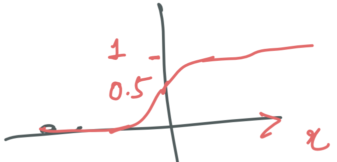
https://playground.tensorflow.org/

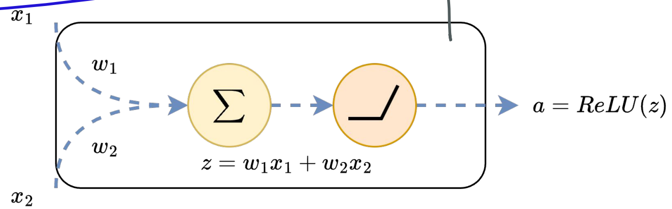
Non-linearities

$\mathbb{R}$

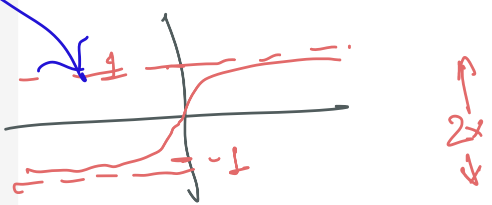
- ReLU $g(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

- Sigmoid $g(x) = \dfrac{1}{1+e^{-x}}$





- Tanh $g(x) = \tanh(x)$



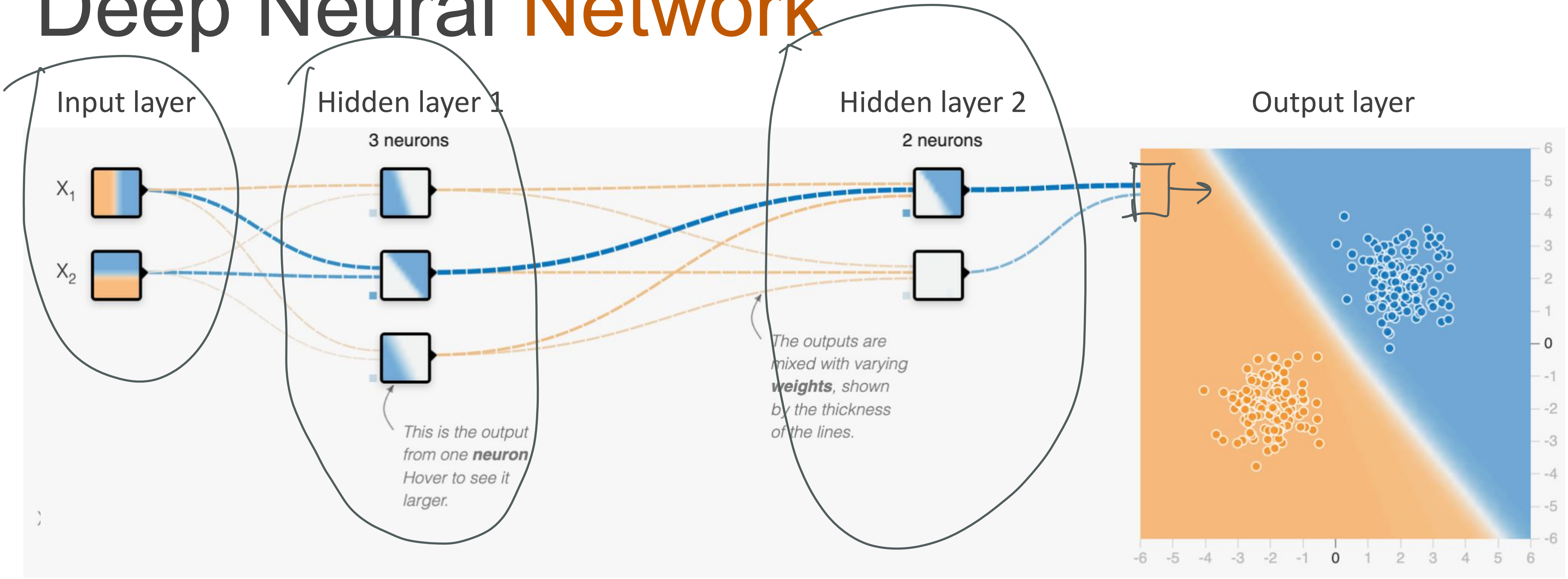
- Softmax $g(x)_i = \dfrac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$

$x \in \mathbb{R}^n$ $\quad g: \mathbb{R}^n \to \mathbb{R}^n$

$$\boxed{\begin{gathered}\sum_{i=1}^{n} g(x)_i = 1 \\ g(x)_i > 0\end{gathered}}$$

softmax

Deep Neural Network



A layered arrangement of neurons form a neural network

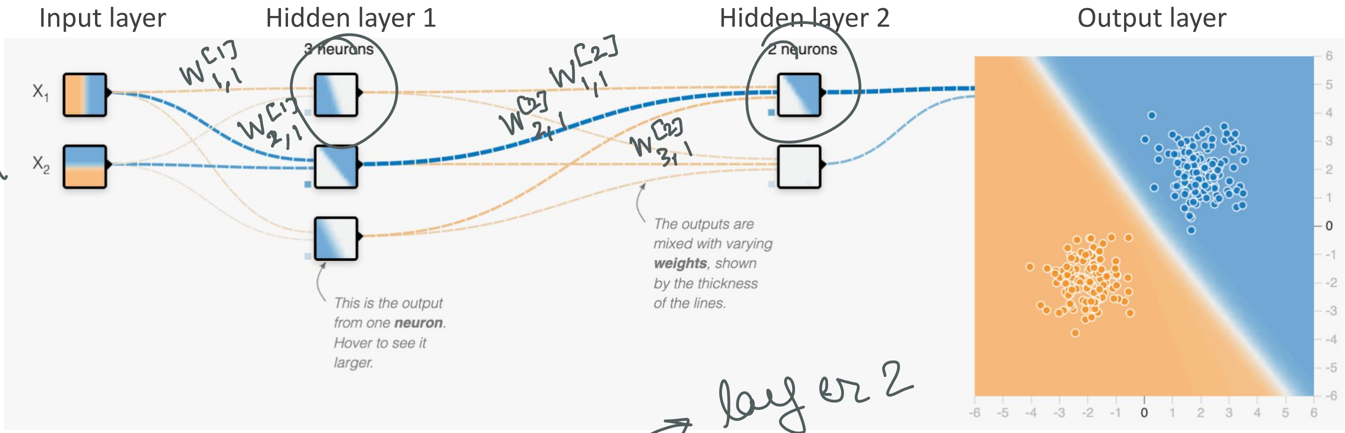
https://playground.tensorflow.org/

Deep Neural **Network**

$H^{[1]}_{1}$ → layer 1

1 → index of this neuron

$$\begin{bmatrix} W^{[1]}_{1,1} \\ W^{[1]}_{2,1} \end{bmatrix}$$



$H^{[2]}_{1}$ → layer 2

1 → index of this neuron

$$\begin{bmatrix} W^{[2]}_{1,1} \\ W^{[2]}_{2,1} \\ W^{[2]}_{3,1} \end{bmatrix}$$

https://playground.tensorflow.org/

batch-size

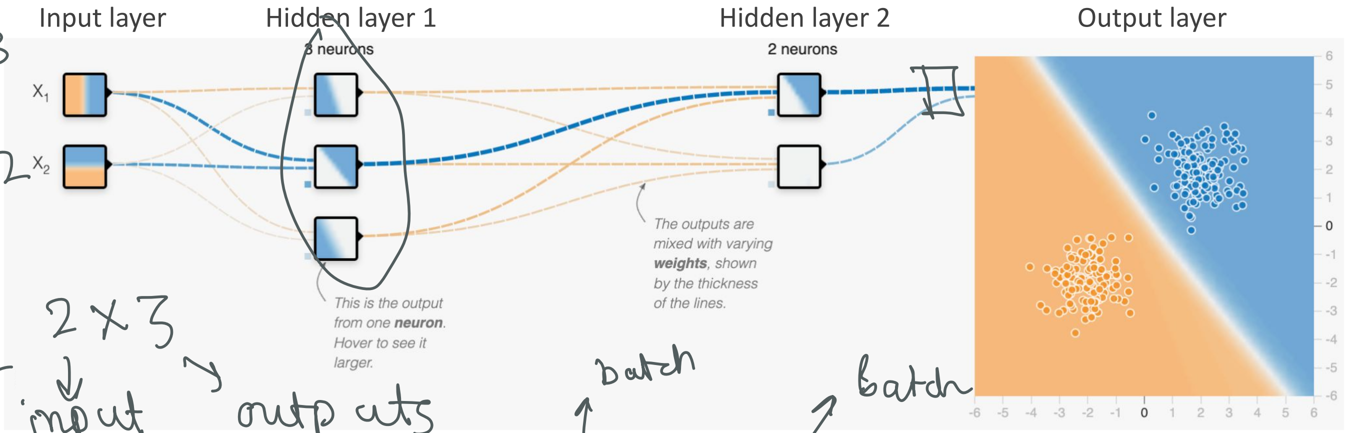
Matrix Representation

$X \in \mathbb{R}^{2\times 1}$

$H^{[1]} \rightarrow W^{[1]} \in \mathbb{R}^{2\times 3}$

$H^{[2]} \rightarrow W^{[2]} \in \mathbb{R}^{3\times 2}$

$O \rightarrow W^{[3]} \in \mathbb{R}^{2\times 1}$



2×3

input outputs

batch

batch

$Z^{[1]} = W^{[1]^T} X \in \mathbb{R}^{3\times 1}$

$A^{[1]} = \sigma(Z^{[1]}) \in \mathbb{R}^{3\times 1}$

$Z^{[2]} = W^{[2]^T} A^{[1]} \in \mathbb{R}^{2\times 1} \rightarrow$ batch

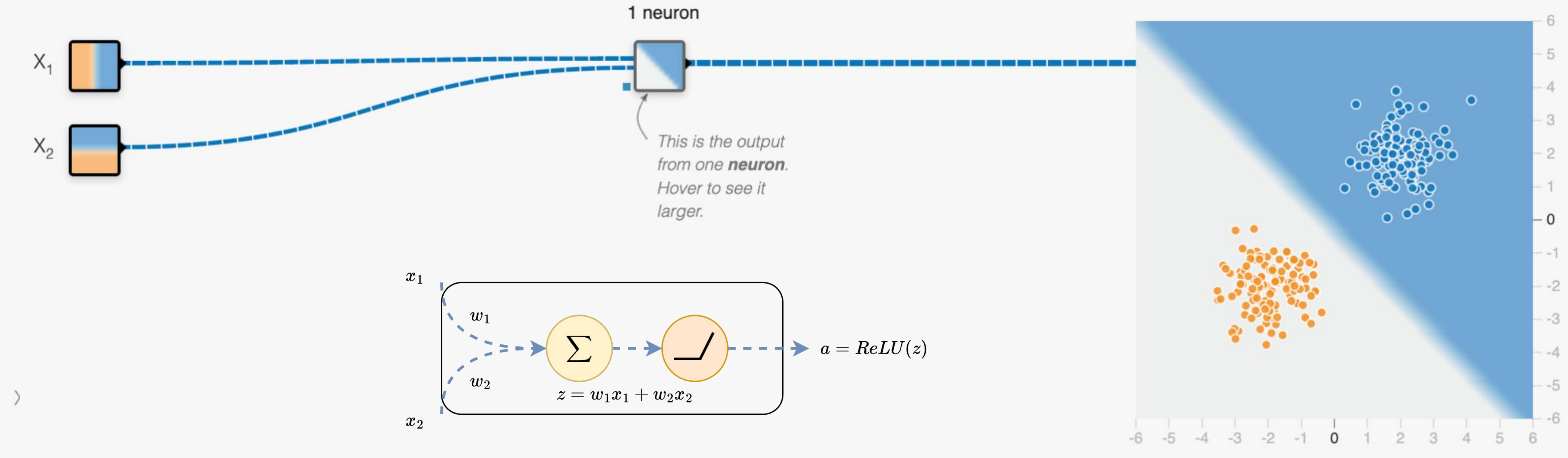
$A^{[2]} = \sigma(Z^{[2]})$

$Z^{[3]} = W^{[3]^T} A^{[2]} \in \mathbb{R}^{1\times 1} \rightarrow$ batch

$A^{[3]} = \sigma(Z^{[3]}) \in \mathbb{R}^{1\times 1}$

https://playground.tensorflow.org/

Why Non-linearity?



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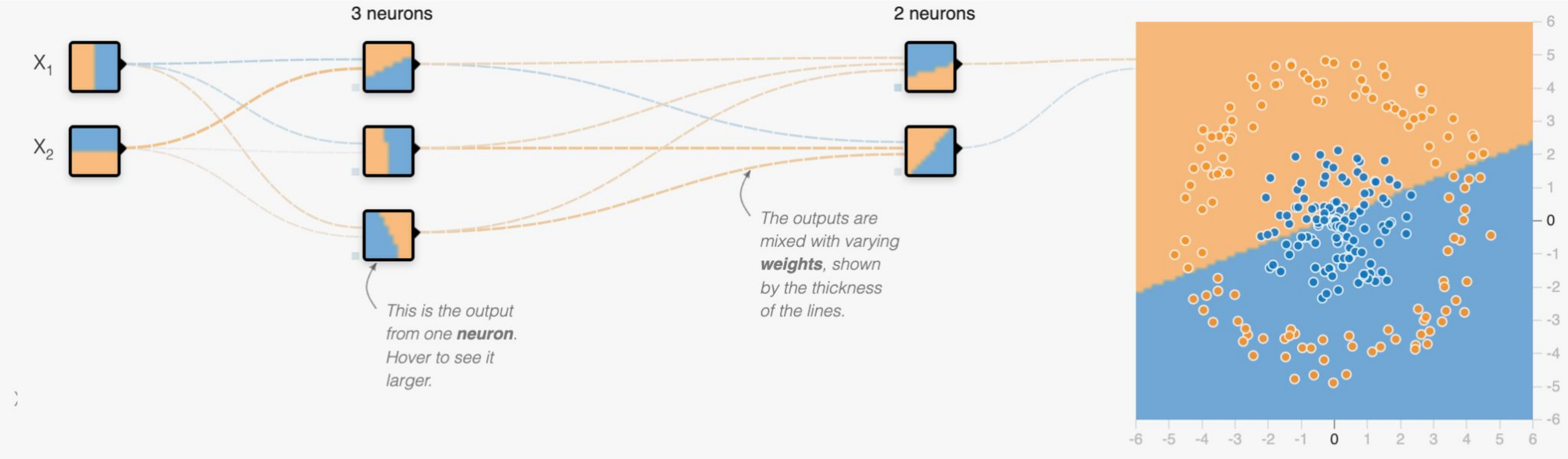
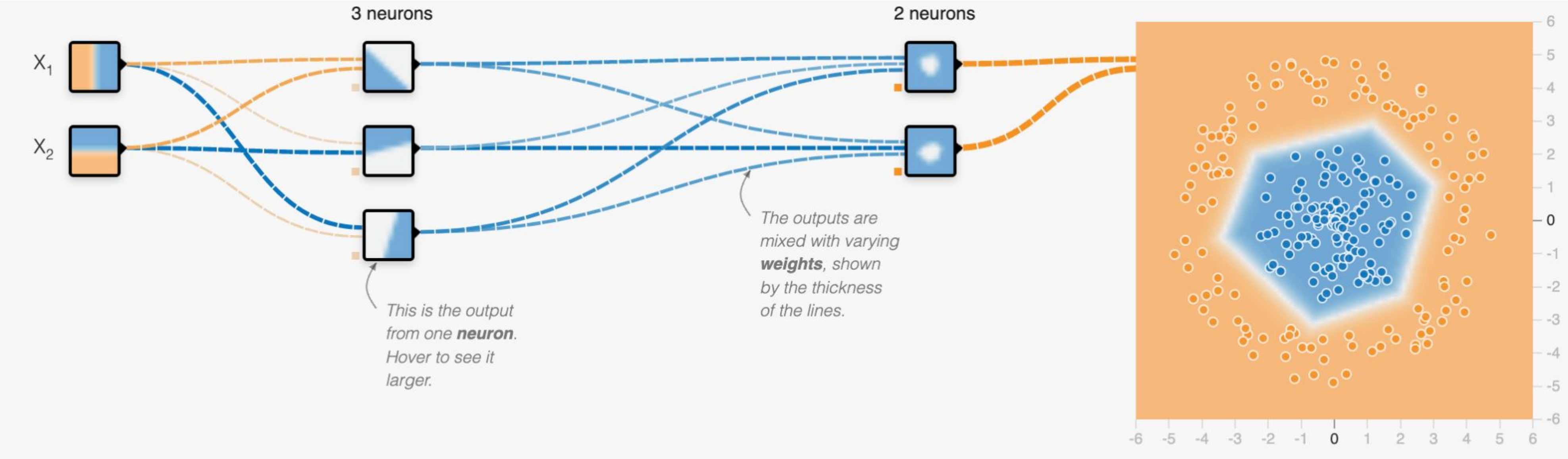
Why Non-linearity?



Combination of linear neurons is still linear

https://playground.tensorflow.org/ (link)

Why Non-linearity?



Why?

Combination of non-linear neurons is much more expressive!

https://playground.tensorflow.org/ (link)

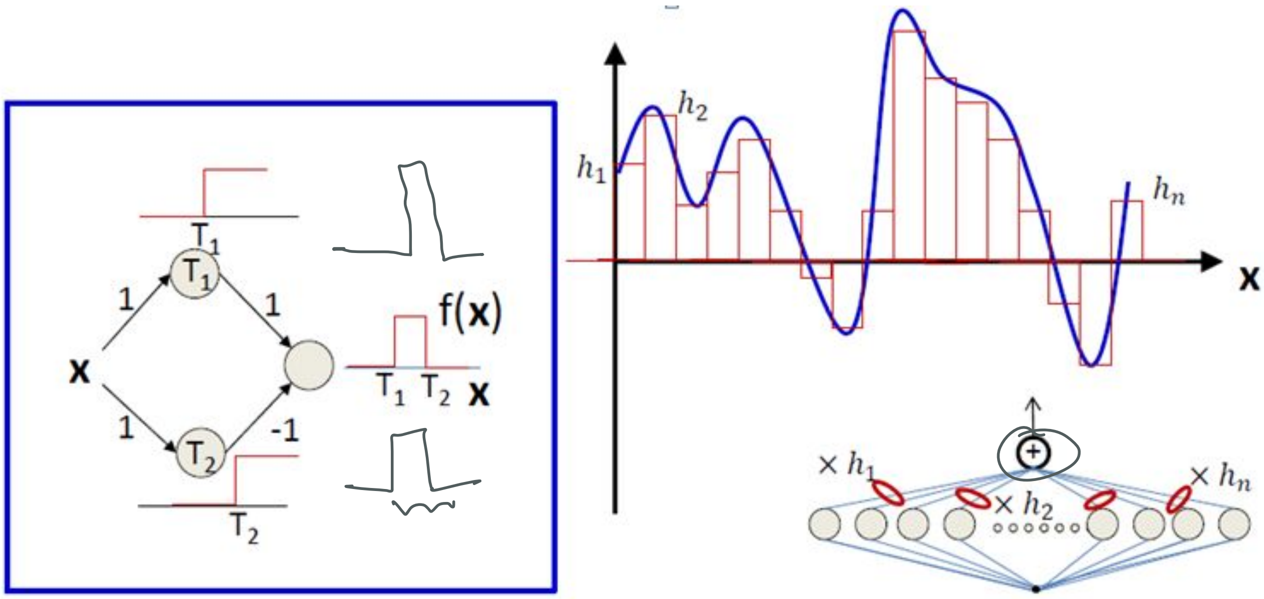
Universal Approximation Theorem

A neural network with

- at least one hidden layer,
- using a sufficiently large number of neurons and,
- an appropriate activation function

can approximate any continuous function on a compact subset of reals to any desired degree of accuracy.

Universal Approximation Theorem



Two neurons can be constructed to give a pulse function

https://medium.com/analytics-vidhya/neural-networks-and-the-universal-approximation-theorem-e5c387982eed

Loss Functions

Loss

- A measure of how “good” our model is for an input
- Usually quantified as the cost of an incorrect prediction

Empirical Loss

The total loss averaged over entire dataset

$n \rightarrow$ data points

$$\mathcal{L} = \frac{1}{n} \sum_{i=1}^{n} l(\theta, x^{(i)})$$

Empirical loss | risk.

Loss Function MSE

Mean Squared Error (MSE)

$$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

$\mathbb{R}$

$$\ell = (\hat{y}_i - y_i)^2$$

$\mathbb{R}$

Loss Function BCE

Binary Cross Entropy (BCE)

Logistic regression

$$\mathcal{L} = -\frac{1}{N}\sum_{i=1}^{N}\left(y_i \log \hat{y}_i + (1 - y_i)\log(1 - \hat{y}_i)\right)$$

$y_i = \{0, 1\}$

$\hat{y}_i = P(\text{class being present}) \in (0, 1)$

Loss Function CE

Generalization to C class classification problem

Cross Entropy (CE)

$$\mathcal{L} = -\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{C} y_{ij}\log \hat{y_{ij}}$$

C classes to classify into

$C=2 \rightarrow$ BCE

$$\hat{y}_i = \underbrace{[\qquad\qquad]}_{\text{probability dist.}} \in \mathbb{R}^C$$

probability dist. $\rightarrow$ softmax

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Example Task 1

Input

https://www.megavoxels.com/learn/how-to-make-a-pixel-art-cat/

Task - is there a cat in the image?

Loss - BCE

Example Task 2

Input

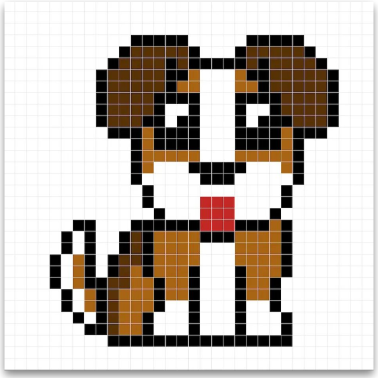
https://www.megavoxels.com/learn/how-to-make-a-pixel-art-cat/

Task - determine the age of the cat?

Loss - MSE

Example Task 3

Input



Task - determine cat vs dog vs bird?

Loss - CE softmax

3 - way

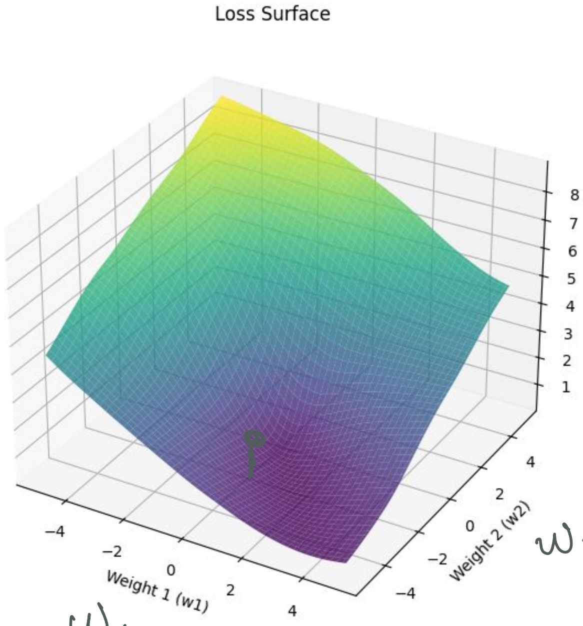
https://www.megavoxels.com/learn/how-to-make-a-pixel-art-cat/

Optimization

Gradient Descent & Backprop

Optimization Overview

Minimize the loss by adjusting the model parameters



loss

$= L(w_1, w_2, D)$

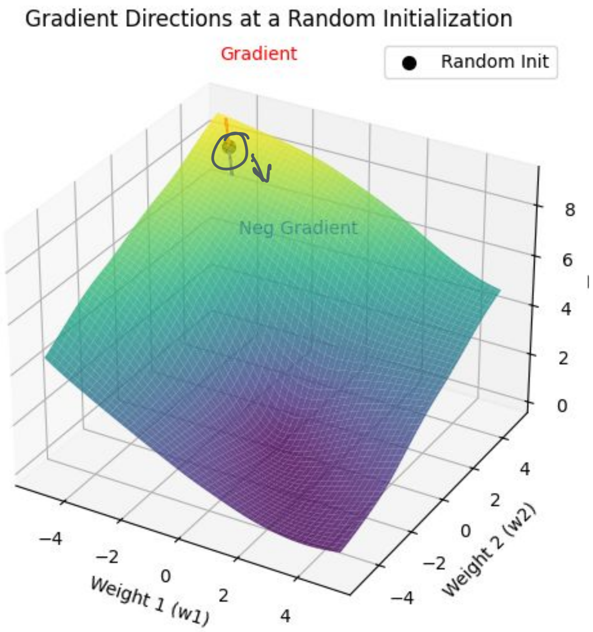
$\hookrightarrow$ scalar

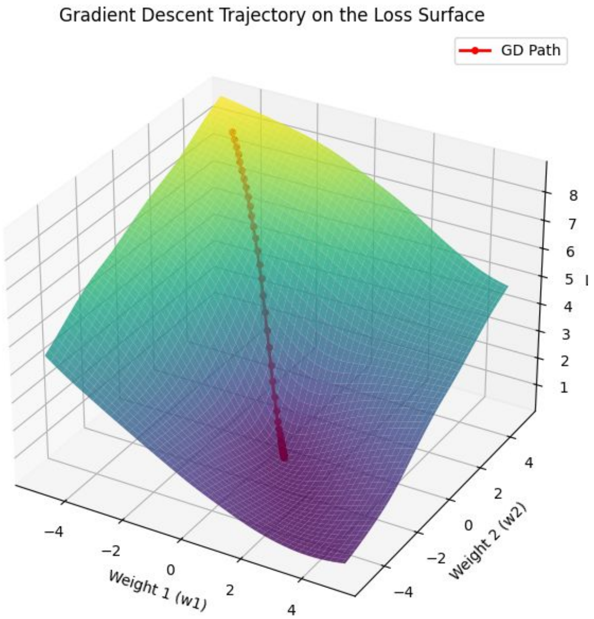
w_2

w_1

Gradient Descent

Iteratively update parameters in opposite direction of the gradient





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Gradient Descent

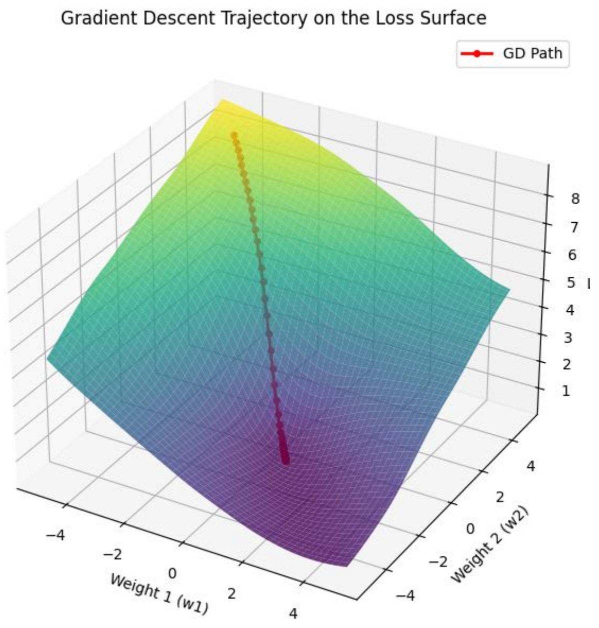
Initialize parameters (weights w)

For each iteration

1. Compute the loss L
2. Compute gradients dL/dw
3. Update parameters w = w - η * (dL/dw)

where η is the learning rate

scale



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Gradient Descent

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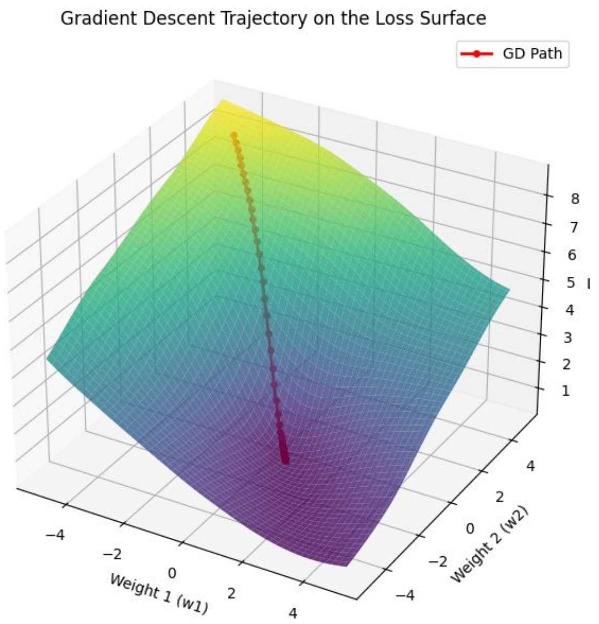

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Backpropagation

- Neural networks require an efficient way to compute the gradients for all parameters
- **Backpropagation** - chain rule to propagate errors from the output back to the input layer

Backpropagation

Chain Rule - for a parameter θ that affects the loss $\mathcal{L}$ through an intermediate variable z

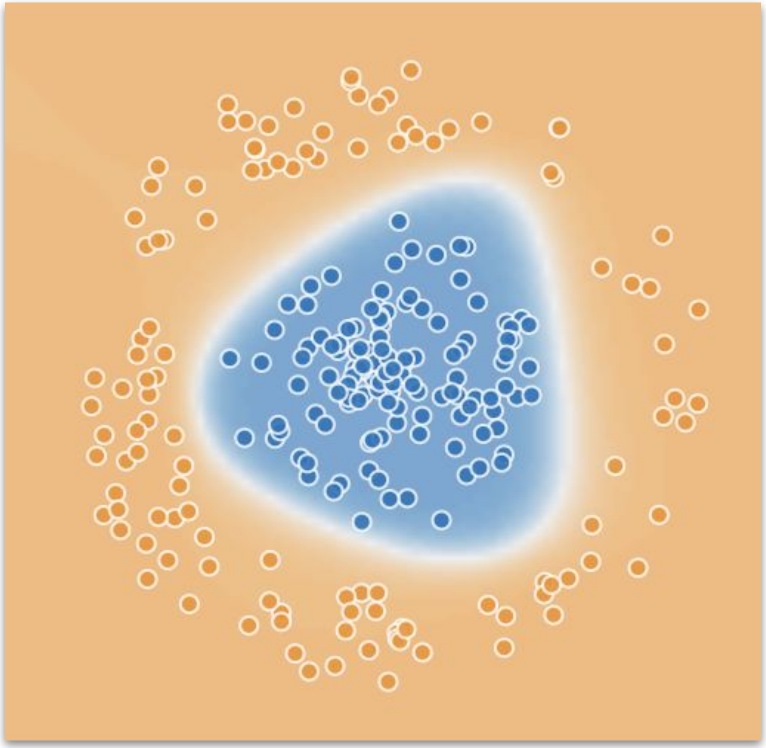
$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial z} \cdot \frac{\partial z}{\partial \theta}$$

Forward Pass - Compute all intermediate activations $Z^{[l]}$ and $A^{[l]}$

Backward Pass - Starting at the output layer, compute the gradient and propagate it backwards

Sample DL Algorithm

Task



Task - classify orange from blue

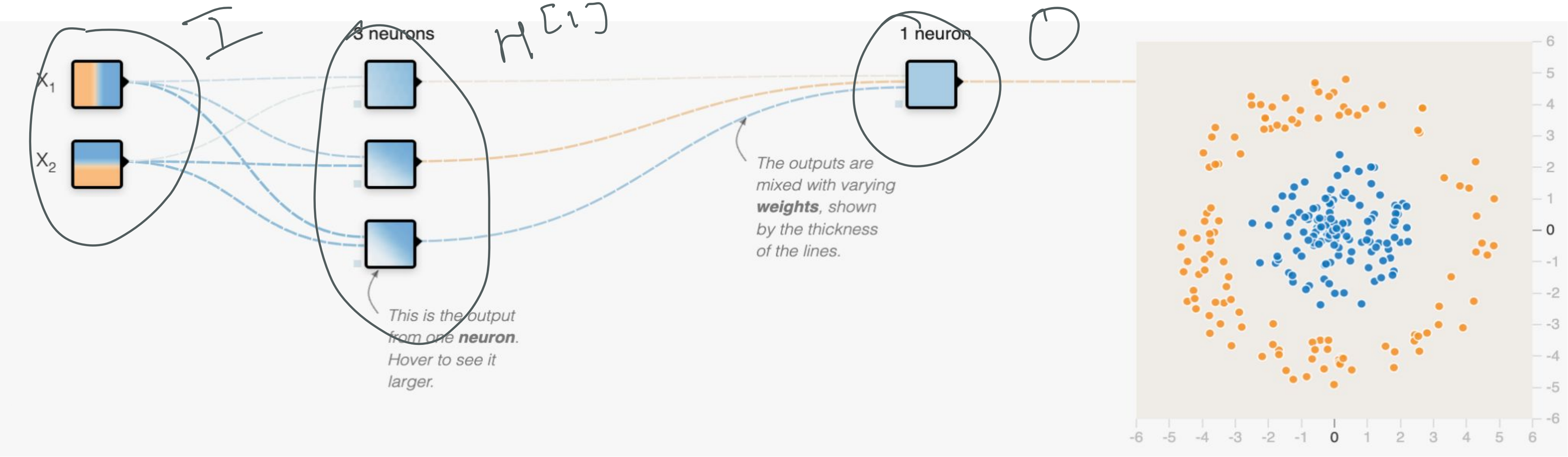
Loss

Binary Cross Entropy (BCE)

$$\mathcal{L} = -\frac{1}{N}\sum_{i=1}^{N}\left(y_i \log \hat{y}_i + (1 - y_i)\log(1 - \hat{y}_i)\right)$$

Model

Sigmoid activation, 3 hidden neuron, 1 output neuron



Input layer Hidden layer Output layer

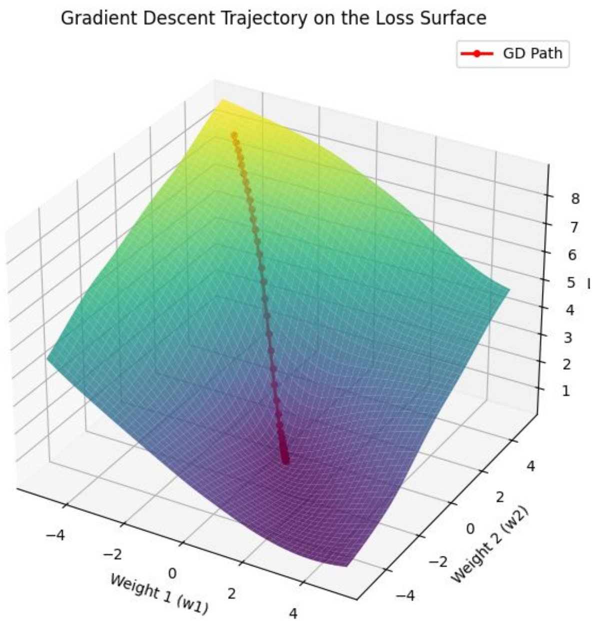
Optimization

Initialize parameters (weights w)

For each iteration

1. Compute the loss L
2. Compute gradients dL/dw
3. Update parameters w = w - η * (dL/dw)

where η is the learning rate.



Optimization - Forward Pass & Loss

$X \in \mathbb{R}^{2\times 1}$

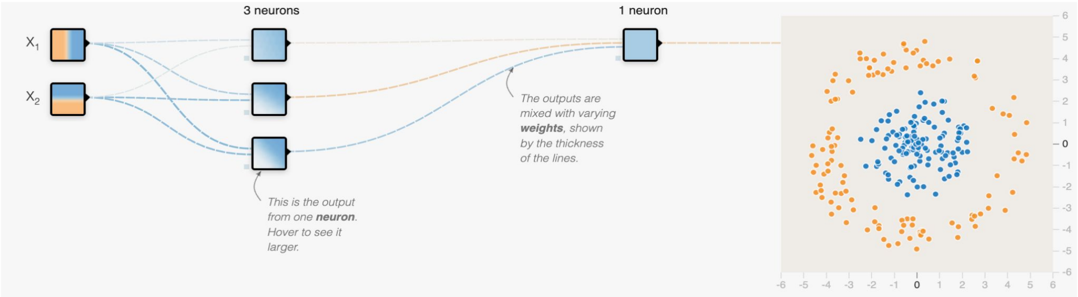
$H^{[1]} \rightarrow W^{[1]} \in \mathbb{R}^{3\times 2}$

$O \rightarrow W^{[2]} \in \mathbb{R}^{1\times 3}$

$Z^{[1]} = W^{[1]T} X \; ; \; A^{[1]} = \sigma(Z^{[1]})$

$Z^{[2]} = W^{[2]T} A^{[1]} \; ; \; A^{[2]} = \hat{Y} = \sigma(Z^{[2]})$

$\mathcal{L} = -(Y \cdot \log(\hat{Y}) + (1-Y) \cdot \log(1-\hat{Y}))$



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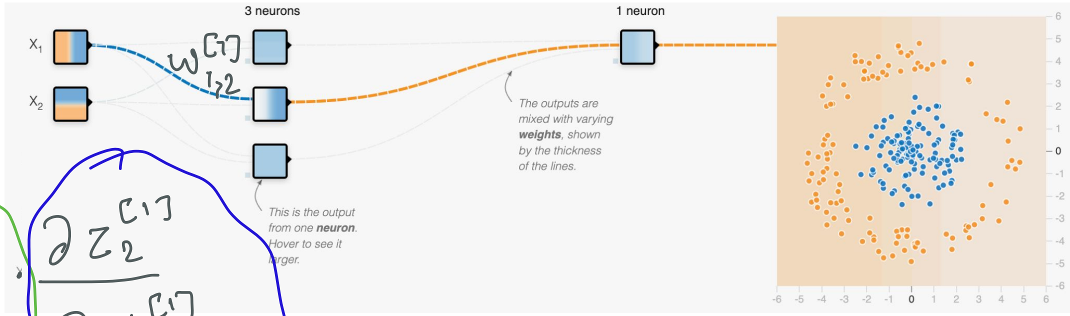
Optimization - Backward Pass

$$\boxed{\text{Compute } \frac{\partial L}{\partial w^{[1]}_{1,2}}}$$



Chain Rule

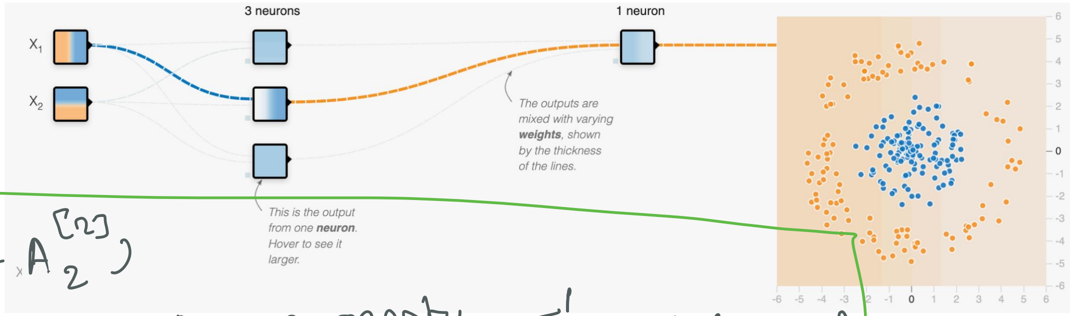
$$\frac{\partial L}{\partial w^{[1]}_{1,2}} = \frac{\partial L}{\partial Z^{[2]}} \cdot \frac{\partial Z^{[2]}}{\partial A^{[1]}_2} \cdot \frac{\partial A^{[1]}_2}{\partial Z^{[1]}_2} \cdot \frac{\partial Z^{[1]}_2}{\partial W^{[1]}_{1,2}}$$

Forward

$$L = Y \log(\hat{Y}) + (1-Y)\log(1-\hat{Y}) \quad ; \quad \hat{Y} = \sigma(Z^{[2]})$$

$$\Rightarrow \frac{\partial L}{\partial Z^{[2]}} = (\hat{Y} - Y) \quad \text{(after simplifying)}$$

Optimization - Backward Pass



forward

$$Z^{[2]} = W^{[2]T} A^{[1]} \Rightarrow \frac{\partial Z^{[2]}}{\partial A_2^{[1]}} = W_2^{[2]}$$

$$A_2^{[2]} = \sigma(Z_2^{[1]}) \Rightarrow \frac{\partial A_2^{[2]}}{\partial Z_2^{[1]}} = A_2^{[2]} \cdot (1 - A_2^{[2]})$$

$\hookrightarrow$ using property $\sigma' = \sigma \cdot (1 - \sigma)$

$$Z^{[1]} = W^{[1]T} X \Rightarrow Z_2^{[1]} = W_{1,2}^{[1]} X_1 + W_{2,2}^{[1]} X_2 \Rightarrow \frac{\partial Z_2^{[1]}}{\partial W_{1,2}^{[1]}} = X_1$$

$$\frac{\partial L}{\partial W_{1,2}^{[1]}} = (\hat{Y} - Y) \cdot W_2^{[2]} \cdot A_2^{[1]}(1 - A_2^{[1]}) \cdot X_1$$

$\hookrightarrow$ Final gradient for $W_{1,2}^{[1]}$

Full Algorithm

- **Step 1 (Forward Pass)**: Compute activations
- **Step 2 (Loss Computation)**: Evaluate the loss using the predicted probability and the true probability
- **Step 3 (Backward Pass)**: Use backpropagation to compute gradients for all parameters
- **Step 4 (Parameter Update)**: Adjust model weights using gradient descent

Repeat for multiple steps until convergence

In Practice

- **Efficiency** - mini-batch gradient descent
- **Learning Rate** - adaptive learning rate gradient descent
- **Regularization** - dropout, weight decay, early stopping, etc